Metrological analysis of ancient buildings in Persepolis

Non-invasive methods in the contemporary archaeological practice

Paweł Borycki

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http://www.pborycki.pl/pdf/modul5_eng.pdf

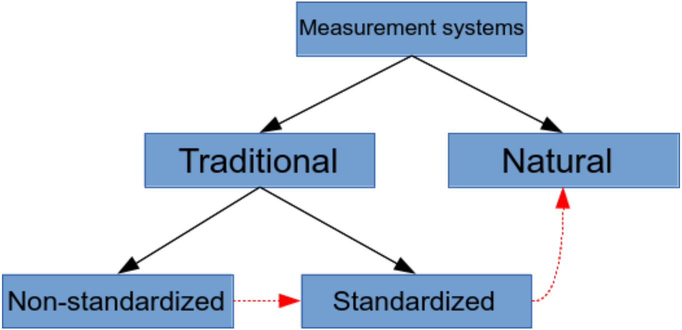
Plan of the presentation

1. Length and its measure
   - Systems of measurement
   - Association with numeral system
     - Regular numbers
     - Middle Eastern numeral systems
   - Sumerian-Babylonian length measurement system
   - Proposed length measurement systems for Persepolis
2. Architectural module
   - Definition
   - Methods of computation
     - Classes of architectural dimensions
     - Cosine quantogram method / Modified least squares method
   - Sample computation
     - Palace of Xerxes in Persepolis

Phases of development of measurement systems

- **Gordon V. Childe** – 3 phases of measurement systems development
- Increase in importance of craftsmanship and trade
  - Formalization of definitions of measurement units
  - Precision of measurement
- **1. Measurement by comparison and binary decision**
  - Production for the own individual household
  - Lack of specialization
  - Eg. fitting parts of the tool
- **2. Traditional length measurement system**
  - Units based on length of body parts
  - Length described as a number of units
  - Exchange of goods between the households
- **3. Standardized traditional length measurement system**
  - Precision and repeatability of length units
  - Standardization of units performed by elites
  - Presence of prototypes, eg. measuring rods

Classification of measurement systems



- Natural systems (eg. **metric**)
  - Defined by the physical constants
- Traditional standardized systems (eg. **imperial**)
  - Defined by the law

- Larger units
  - Multiples of smaller units
    - **Regular numbers**

Regular numbers

- Positional numeral system with a base (radix) x
  - $(a_i \dots a_1 a_0)_x = a_0 \cdot x^0 + a_1 \cdot x^1 + \dots + a_i \cdot x^i$
  - $0 \le a_0 \dots a_i \le x - 1, a_0 \dots a_i \in \mathbb{N}$
- Old Babylonian period
  - Sexagesimal numeral system
  - Numbers containing only 2, 3 and 5 in their prime factorization

Definition (Regular number in positional numeral system)

A **regular number** in numeral positional system with a base x is:

1. An integer number whose prime factors form a subset of the set of all prime factors of the number x,
2. A fraction which satisfies the condition 1 after being multiplied by some power of x with integer exponent.

Regular numbers

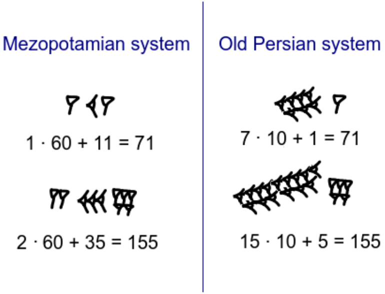
- Examples of regular numbers
  - Decimal system
    - **4, 25,** $\frac{5}{2}$
    - **3, 15,** $\frac{1}{6}$
- Property
  - Every regular number in numeral system with base *x* has a **finite** positional notation in this system
- Regular measurement system in numeral system with base *x*
  - Proportion of values of **every two units** of measurement system is a regular number in the numeral system with base *x*
- Examples of regular measurement systems
  - Metric system in decimal numeral system
  - Imperial system in decimal numeral system
    - **1 feet = 12 inches =** $2^2 \cdot$ **3 inches**

Mesopotamian numeral system

Double digits

- Additive-positional system
  - Sexagesimal
  - *Double digits*
  - Lack of decimal mark
    - **Nominal and real values (17, $\frac{17}{60}$, $17 \cdot 3600$)**
- Development of the numeral system
  - Early Dynastic Period (c. 2900–2400 BC)
    - **Creation of the system**
  - Old Babylonian Period (c. 1894–1595 BC)
    - ***Zero symbol or space***
  - Seleucid Dynasty (312–141 BC)
    - **Leading and trailing zeros in astronomical texts**

Middle Eastern numeral systems



Number notation in Mesopotamian and Old Persian numeral systems

- Mesopotamian mixed decimal system
  - Rules of notation
    - $< 100 \rightarrow$ **sexagesimal system**
    - $\geq 100 \rightarrow$ **decimal system**
  - *13 li-im 9 mi-at 1,12*
    - **Abu Salabich, Early Dynastic Period**
  - $13 \cdot 1000 + 9 \cdot 100 + (1, 12)_{60} = 13000 + 900 + 1 \cdot 60 + 12 \cdot 1 = 13972$
- Old Persian System
  - Additive decimal system
  - Mesopotamian *double digits*
    - **Unlimited value of “digits”**
    - **Lack of positional notation**

Relations between units

- **1.** Every larger unit is a **multiple** of a smaller unit
  - Imperial system
- **2.** Proportions of lengths of all units are **regular numbers** in the numeral system used by the culture considered
  - Standard Sumerian-Babylonian system
- **3.** Proportions of lengths of all units are **powers with integer exponent** of the base of the numeral system used by the culture considered
  - Metric system

$$\mathbf{1} \wedge \mathbf{2} \leftarrow \mathbf{3}$$

Sumerian-Babylonian length measurement system

Unit name	**Sumerian name**	**Akkadian name**	**Length in *ammatu***	**Length [cm]**
Barley	še	*uṭṭetu*	$\frac{1}{180}$	0,278
Finger	šu-si	*ubānu*	$\frac{1}{30}$	1,667
$\frac{1}{3}$ *Cubit*	šu-dù-a	*šīzu / šizû*	$\frac{1}{3}$	16,667
Half Cubit	zipaḫ	*ūṭu*	$\frac{1}{2}$	25
Cubit	kùš	*ammatu*	1	50
Half Reed	—	*nikkas / nikkassu*	3	150
Reed	gi	*qanû*	6	300
Rod	—	*nindan / nindānu*	12	600
Half Rope	—	*ṣuppu*	60	3 000
Rope	éše	*ašlu*	120	6 000
"UŠ"	—	—	720	36 000
"bēru"	dana / danna	*bēru*	21 600	1 080 000

Standard Sumerian-Babylonian length measurement system

Sumerian-Babylonian length measurement system

- Regular system in sexagesimal numeral system
- Interpretation of the notation "***3,20 ammatu***"
  - Value **in the order of** *ammatu*, which has a nominal value equal (3,20) *nindan* (1 *ammatu* = $\frac{1}{12}$ *nindan*)

$$3 \cdot 60^{x+1} + 20 \cdot 60^{x} = \frac{1}{12}\ [nindan]$$

$$60^{x} = \frac{1}{4000}$$

$$x = \log_{60}\left(\frac{1}{4000}\right) = -2,025733 \approx -2$$

$$(0;03,20)_{60} = 3 \cdot 60^{-1} + 20 \cdot 60^{-2} = \frac{3}{60} + \frac{20}{3600} = \frac{9+1}{180} =$$

$$= \frac{1}{18}\ [nindan] = \frac{2}{3}\ [ammatu].$$

System of Petrie (1877)

- Drawings made by Pascal Coste and Eugène Flandin (1840)
- Regularity
  - Larger units are not multiples of the smaller units
  - Lack of regularity in sexagesimal numeral system
- Association with numeral systems
  - Multiples of basic unit the most frequent
    - **25, 100 – regular in decimal numeral system**
    - $\frac{40}{3}$, $\frac{50}{3}$ **– regular in sexagesimal numeral system**
  - Arish (India), royal cubit (Egypt) – 64,325 cm (25 $\times$ BU)

Unit name	Length [cm]	Length [inch]
$\frac{1}{50}$ *Arish*	**1,944**	0,7652
(Basic Unit)	**2,573**	1,013
$\frac{1}{12}$ *Babylonian cubit*	**4,394**	1,730
Babylonian cubit	**52,578**	20,700
Assyrian cubit	**54,293**	21,375

Length measurement system for Persepolis proposed by Petrie (1877)

System of Babin (1891)

- Drawings made by Pascal Coste and Eugène Flandin (1840)
- Regularity
  - Regular system in sexagesimal numeral system
  - $\frac{\text{Thumb}}{\text{Finger}} = \frac{\text{Foot}}{\text{Cubit}} = \frac{5}{3}$
- Association with Mesopotamian systems
  - Lack of units of similar length
- Module as a radius of column measured at fixed height
  - **Xerxes Gate** – 15,5 fingers = 71,3 cm
  - **Apadana** – 17 fingers = 78,2 cm

Unit name	Length [cm]	Length [cubit]	Length [thumb]
Thumb (pouce)	2,75	$\frac{1}{20}$	1
Finger (doigt)	4,6	$\frac{1}{12}$	$\frac{5}{3}$
Foot	33	$\frac{3}{5}$	12
Cubit	55	1	20

Length measurement system for Persepolis proposed by Babin (1891)

System of Krefter (1971)

- Dimensions of bricks
  - Mud bricks – 33 × 33 × 13 cm
  - Thickness of joints – 1,0-1,5 cm
- Regularity
  - Regular system in sexagesimal numeral system
  - Finger, palm, foot, cubit
    - **Equivalents in Mesopotamian systems**
    - **r = 1,0272, if *ammatu* = 50 cm**
  - Royal cubit
    - **Twice the length of foot, computational procedure?**

Unit name	Length [cm]	Length [cubit]	Length [finger]
Finger	**2,14**	$\frac{1}{24}$	1
Palm	**8,56**	$\frac{1}{6}$	4
Foot	**34,24**	$\frac{2}{3}$	16
Cubit	**51,36**	1	24
Royal cubit	**68,48**	$\frac{4}{3}$	32

Length measurement system for Persepolis proposed by Krefter (1971)

System of Roaf (1978)

- Markers on platforms of Palace of Xerxes and Palace of Darius
- Intercolumniations
- Regularity
  - Regular system in sexagesimal numeral system
  - Isomorphism with subsets of Mesopotamian systems and the system of Krefter
    - **Lack of royal cubit**
  - Coefficients of proportionality
    - **Mesopotamian – r = 1,0440, if *ammatu* = 50 cm**
    - **Krefter – r = 1,0164**

Unit name	**Length [cm]**	**Length [Pct.]**	**Length [Pfinger]**
Persepolitan finger (Pfinger)	**2,2**	$\frac{1}{24}$	1
Persepolitan palm (Ppalm)	**8,7**	$\frac{1}{6}$	4
Persepolitan foot (Pft.)	**34,7 – 34,8**	$\frac{2}{3}$	16
Persepolitan cubit (Pct.)	**52,1 – 52,2**	1	24

Length measurement system for Persepolis proposed by Roaf (1978)

Length units proposed for Persepolis

- **Lack of preserved texts and measuring devices**
- Regular systems in sexagesimal numeral system
- Coefficient of variation for common units
  - $v = \frac{\sigma}{\mu}, \quad \mu \neq 0$
  - cubit – **3,6%**
  - foot – **2,7%**
  - finger – **14,2%**
    - **Inability to merge units called *finger***

	Babin 1891	Krefter 1971	Roaf 1978
royal cubit	—	68,48	—
cubit	55,20	51,36	52,1 – 52,2
foot	33,10	34,24	34,7 – 34,8
palm	7,43	8,56	8,7
finger	2,75	2,14	2,2

Length units proposed for Persepolis [cm]

Architectural module

"The front of a Doric temples is to be divided into 27 parts if it is tetrastyle, into 42 parts if it is hexastyle. Of these one part will be ***the module*** *and when this is determined, the distribution of all the work is produced by multiples of it."* (Vitruvius)

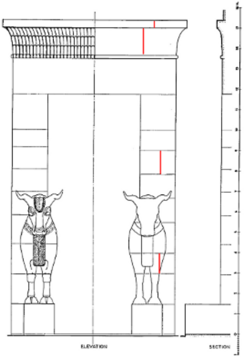
Definition (Module)

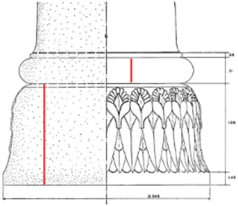
Architectural module of a set of measurements A (M_A) is the largest unit such that every value of the measurement $a \in A$ may be interpreted as a multiple of the module with an absolute error ϵ:

$$\forall . a \in A : \exists . n \in \mathbb{N}_+, \epsilon < M_A : \; a = n \cdot M_A + \epsilon.$$

Module as a special case of length unit.

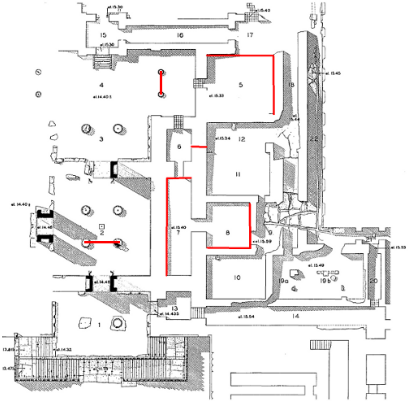
Architectural dimensions of the 1st class

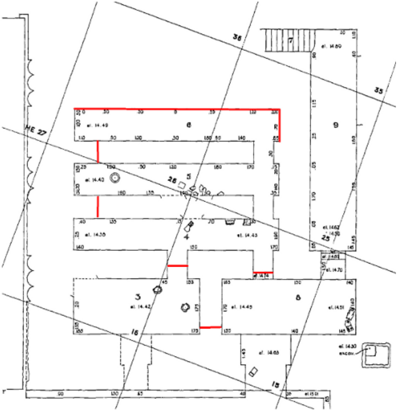




- **Single structural elements**
  - **Bricks, stone blocks**
- **Architectural details**

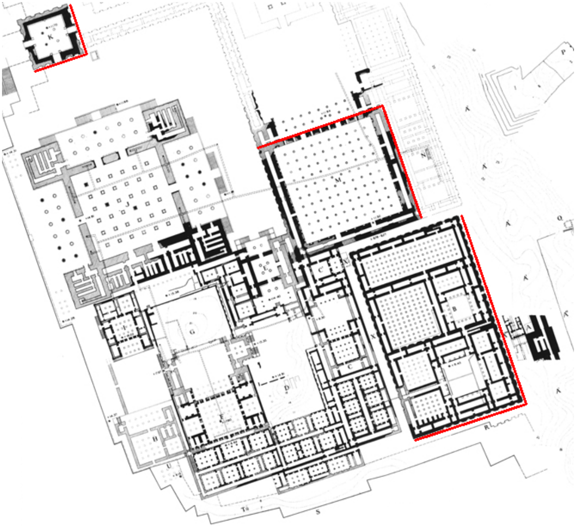
Architectural dimensions of the 2nd class





- **Rooms**
- **Walls**
- **Intercolumniations**

Architectural dimensions of the 3rd class



- **Buildings**
- **Thoroughfares**

Classes of architectural dimensions and a module

- **Differences between computed values**
  - **Different units in the measurement system**
- Dimensions of the 1st class
  - $\rightarrow$ finger ($\sim$ 2 cm)
  - $\rightarrow$ palm ($\sim$ 8 cm)
- Dimensions of the 2nd and 3rd class
  - $\rightarrow$ foot ($\sim$ 34 cm)
  - $\rightarrow$ cubit ($\sim$ 52 cm)
- Vertical and horizontal dimensions
  - Different units? – Different precision?
- **Verification of the computed results**
  - Independent datasets
  - Multiples of the smaller units

Methods of module computation

- Cosine quantogram method

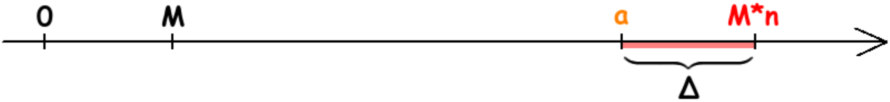
$$\phi(x) = \sqrt{\frac{2}{N}} \sum_{i=1}^{N} \cos\left(\frac{2\pi a_i}{x}\right)$$

  - Kendall, “Hunting quanta”, 1974
- Modified least squares method

$$\psi(x) = \frac{1}{x^2} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \left(x \cdot \mathrm{round}\left(\frac{a_i}{x}\right) - a_i\right)^2$$

  - Mustonen, 2012

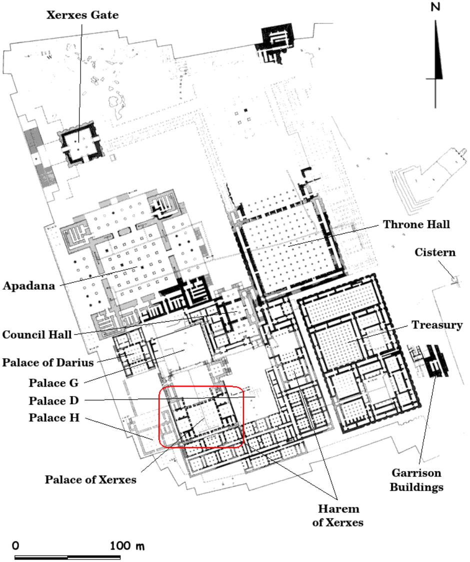
Verification of the computed module



- Assumption: **M** is a module for a set of measurements A
- For every measurement $a \in A$:
  - $n = a/M$ rounded to the nearest integer
  - $a_M = M * n$
  - $\Delta_{(M,a)} = |a_M - a|$
- For all measurements, except for ***negligible number*** of them, $a \in A$ value $\Delta_{(M,a)}$ is ***small***
  - **M** is a module for a set A
- For ***significant number*** of measurements $a \in A$ value $\Delta_{(M,a)}$ is ***large***
  - **M** is not a module for a set A

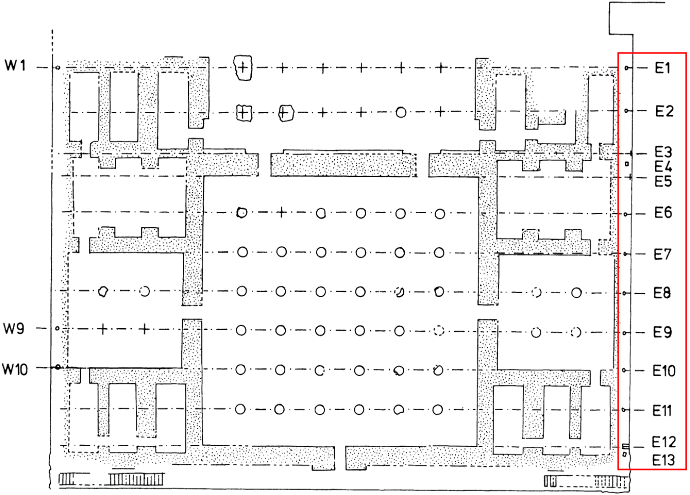
- Statistical evaluation of an error
  - mean, standard deviation, median, minimum, maximum

Palace of Xerxes in Persepolis



- Xerxes I
  - 519 – 465 BC
- Markers in the form of incisions on the platform of the palace
  - **Eastern edge of the platform**
  - Columns, walls, doorways

Module computation – example (ROAF 1978)



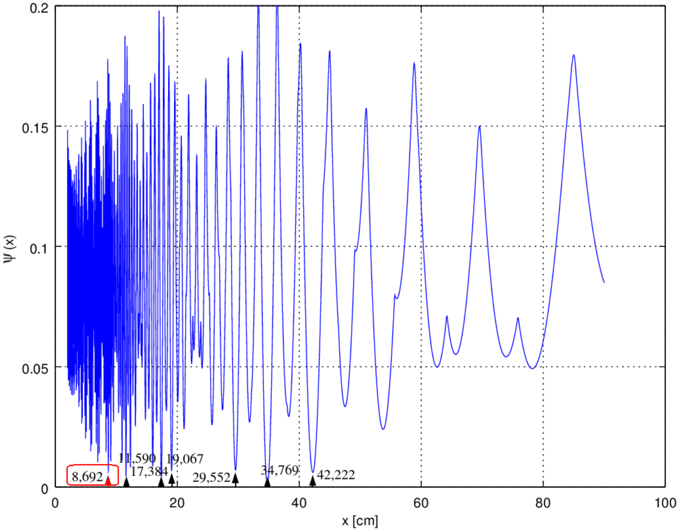
Plan of the Palace of Xerxes (ROAF 1978)

Module computation – example (Roaf 1978)

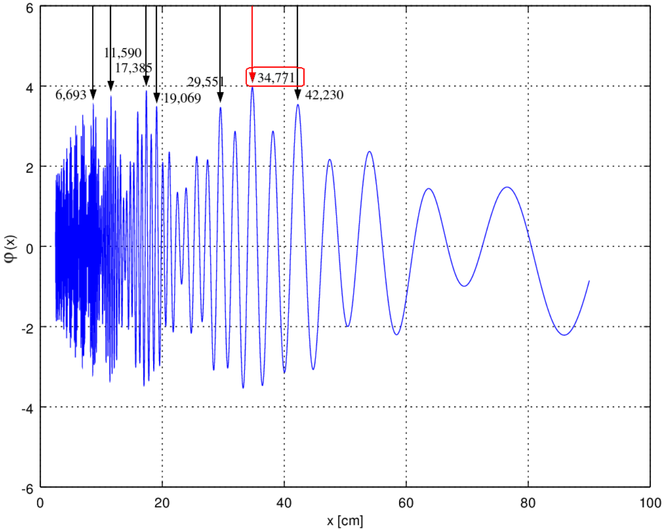
Module	Method of computation	Mean error	Median
4,261	MEDIAN	0,706	0,544
4,684	AVG_TOTAL	1,183	1,328
4,728	MEDIAN	0,907	0,926
5,307	AVG_TOTAL	1,244	1,250
5,225	MEDIAN	0,903	0,788
5,304	AVG_TOTAL	1,254	1,313
5,225	MEDIAN	0,903	0,788
5,313	MEDIAN	1,266	1,250
5,405	AVG_TOTAL	1,318	1,286
5,346	AVG_MINMAX	1,336	1,423
5,457	MEDIAN	1,323	1,250
5,319	MEDIAN	1,276	1,271
5,391	AVG_TOTAL	1,163	0,998
6,830	MEDIAN	0,930	0,674
8,500	AVG_TOTAL	1,065	0,759
8,682	MEDIAN	0,710	0,705
8,659	MEDIAN	1,426	1,432
8,688	MEDIAN	0,594	0,500
8,690	**MEDIAN**	**0,570**	**0,403**
8,685	MEDIAN	0,652	0,602
8,522	MEDIAN	1,279	0,794
13,679	MEDIAN	2,625	1,500
13,696	MEDIAN	2,721	2,000
13,679	MEDIAN	2,625	1,500
13,917	MEDIAN	3,979	5,708
13,688	MEDIAN	2,703	1,750
13,679	MEDIAN	2,625	1,500
19,065	AVG_MINMAX	1,397	1,455
34,772	**MEDIAN**	**0,547**	**0,430**
42,333	MEDIAN	2,729	2,250
42,444	MEDIAN	2,701	1,250
42,342	AVG_TOTAL	2,727	2,172

- **A** = { 417,5; 417,0; 209,0; 382,0; 381,0; 382,5; 383,5; 383,0 } [cm]
- Dimensions of the 2nd class
- Values with the smallest measures of error
  - **Foot = 34,772 cm** (34,760)
  - **Palm = 8,690 cm** (8,700)

Palace of Xerxes – Modified least squares method



Palace of Xerxes – Cosine quantogram method



Verification of the module – example (ROAF 1978)

- **Eastern edge of the Palace of Xerxes platform, Persepolis**
- **A** = { 417,5; 417,0; 209,0; 382,0; 381,0; 382,5; 383,5; 383,0 } [cm]
- Hypothesis:
  - MR = **34,760 [cm]** is a module (ROAF 1978)
  - MB = **34,771 [cm]** is a module (BORYCKI 2013)

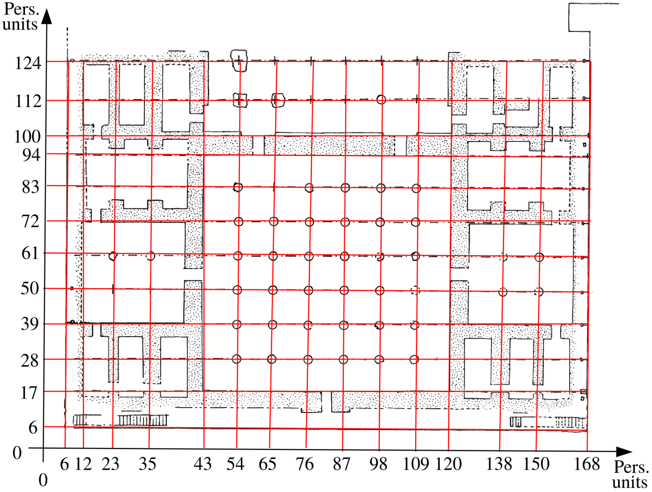
a	n	a_{MR}	a_{MB}	$\Delta_{(MR,a)}$	$\Delta_{(MB,a)}$
417,5	12	417,120	417,264	0,380	**0,236**
417,0	12	417,120	417,264	**0,120**	0,264
209,0	6	208,560	208,632	0,440	**0,368**
382,0	11	382,360	382,492	**0,360**	0,492
381,0	11	382,360	382,492	**1,360**	1,492
382,5	11	382,360	382,492	0,140	**0,008**
383,5	11	382,360	382,492	1,140	**1,008**
383,0	11	382,360	382,492	0,640	**0,508**

Verification of the module – example (ROAF 1978)

- **Eastern edge of the Palace of Xerxes platform, Persepolis**
- **A** = { 417,5; 417,0; 209,0; 382,0; 381,0; 382,5; 383,5; 383,0 } [cm]
- Hypothesis:
  - MR = **34,760 [cm]** is a module (ROAF 1978)
  - MB = **34,771 [cm]** is a module (BORYCKI 2013)

	ROAF 1978	BORYCKI 2013
Mean absolute error	0,573	**0,547**
Mean square error	0,508	**0,500**
Median of error	**0,410**	0,430
Maximum error	**1,360**	1,492
Minimum error	0,120	**0,008**

Palace of Xerxes – Local coordinate system



Plan of the Palace of Xerxes (Roaf 1978)

Metrological analysis of ancient buildings in Persepolis

Thank you.

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